7. A. V. Lykov, Theory of Drying [in Russian], Gosénergoizdat, Leningrad–Moscow (1950).
8. L. M. Nikitina, Thermodynamic Parameters and Mass Transfer Coefficients in Moist Materials [in Russian], Énergiya, Moscow (1968).
9. L. V. Chistotinov, "Some questions on the thermodynamics of unfrozen water in soils," in: Geocryological Studies, Proc. VSEGINGEO, No. 87 [in Russian], Moscow (1975), pp. 24-39.
10. N. S. Ivanov, Heat and Mass Transfer in Frozen Soils [in Russian], Nauka, Moscow (1969).
11. B. P. Veinberg, Ice [in Russian], Gostekhizdat, Moscow–Leningrad (1940).
12. V. V. Bogorodskii and V. P. Gavrilo, Ice. Physical Properties. Contemporary Methods in Glaciology [in Russian], Gidrometeoizdat, Leningrad (1980).
13. N. Ono, "Specific heat and heat of fusion of sea ice," in: Experimental Studies of Heat Exchange in Frozen Soils [in Russian], Nauka, Moscow (1972), pp. 52-60.
14. P. F. Low, D. M. Anderson, and P. Hoekstra, "Some thermodynamic relationships for soils at or below the freezing point," Wat. Resources Res., 4, No. 2, 379-394 (1968).
15. R. I. Gavril'ev, "Determination of the temperature dependence of specific heat of freezing and thawing soils and the quantity of unfrozen water they contain in a single experiment," in: Methods for Determining Thermal Properties of Soils [in Russian], Nauka, Moscow (1970), pp. 16-24.
16. N. A. Tsytovich, "Toward a theory of the equilibrium state of water in frozen soils," Izv. Akad. Nauk SSSR, 9, No. 5, 493-502 (1945).

A UNIFIED METHOD OF NUMERICAL CALCULATION OF THE CONJUGATE PROBLEM OF HEATING BODIES BY LIQUIDS IN CONCURRENT FLOW AND COUNTERFLOW

V. V. Sapelkin

UDC 536.24

A unified algorithm is proposed for the solution of conjugate problems of heat exchange in one-sided and two-sided heating of solid bodies in concurrent flow and counterflow.

In connection with the development of the parameters of the heat-transfer agents used in modern heat exchangers, it becomes necessary to increase the accuracy of calculation of the temperature fields in their elements. Therefore, it is preferable to solve heat-exchange problems in a conjugate statement. In the creation of heat exchangers this makes it possible to reduce their energy capacity and metal content and to increase their productivity by an average of 10% [1].

The mathematical formulation of the problem in a conjugate statement includes the energy equation for the heat transfer agent and the heat-conduction equation for the solid body, as well as, apart from the usual boundary conditions, the conditions at the surface of the body bathed by the heat-transfer agent (the internal boundary conditions). The semidetailed and the detailed conjugate problems of heat exchange should be distinguished. In the first case the temperature field in the heat-transfer agent is described by a quasi-one-dimensional energy equation and the internal boundary conditions have the form of boundary conditions of the third kind. In the second case, two- and three-dimensional energy equations are considered and the internal boundary conditions are boundary conditions of the fourth kind.

Major progress has now been achieved in the solution of conjugate problems of heat exchange through the use of modern numerical and analytic methods. This pertains primarily to problems of heating of bodies in one-sided and two-sided concurrent flow over them [2-5]. At the same time, methods of solving conjugate problems of heat exchange between solid bodies and heat-transfer agents in counterflow are less well developed. The existing semianalytic methods [6-8] have limited application and, in addition, they reduce the problem to an infinite-

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 49, No. 4, pp. 664-671, October, 1985. Original article submitted August 20, 1984.

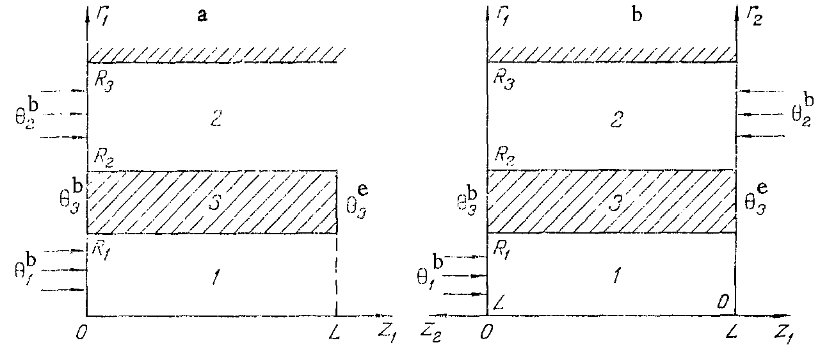

Fig. 1. Diagram of heating of the dividing wall 3 by the heat-transfer agents 1 and 2 in concurrent flow (a) and counterflow (b).

dimensional system of algebraic equations, the numerical solution of which is comparable in complexity with the direct numerical solution of the problem. The application of numerical algorithms, successfully used in the solution of conjugate problems with one-sided heating, to the solution of the problem of heating of bodies by heat-transfer agents moving in counterflow leads to the necessity of iterations, even if the original algorithm is noniterative.

In the present article we consider a unified economical method of numerical solution of conjugate problems of heat exchange, in both the detailed and semidetailed statements, for one-sided and two-sided concurrent flow and counterflow over a solid body by heat-transfer agents, in orthogonal coordinate systems, the coordinate lines of which coincide with the boundaries of the solid body (in this case the temperature field of the heat-transfer agents can be analyzed in other coordinate systems). The method is developed for the case when heat transfer by heat conduction in the heat-transfer agents in the direction of their flow can be neglected.

The flow schemes and the cylindrical coordinate systems used for determinacy are presented in Fig. 1. The coordinate system (r_1, z_1) is used to describe the heating of the dividing wall 3 by the heat-transfer agents 1 and 2 in concurrent flow (see Fig. 1a), while in the case of counterflow (see Fig. 1b) the heat-transfer agent 1 and the wall 3 are analyzed in the coordinate system (r_1, z_1) while the heat-transfer agent 2 is analyzed in the coordinate system (r_2, z_2), with $r_1 = r_2 = r$ and $z_1 = L - z_1$. We analyze the method on the example of the counterflow of heat-transfer agents (see Fig. 1b).

The temperature field in the dividing wall is described by the nonsteady equation of heat conduction

$$\frac{\partial \Theta_3}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Theta_3}{\partial r}\right) + \frac{\partial^2 \Theta_3}{\partial z_1^2}, \qquad (1)$$

$$0 < t \leqslant T, \quad R_1 \leqslant r \leqslant R_2, \quad 0 < z_1 < L.$$

The initial and boundary conditions at the ends of the wall at $z_1 = 0$ and $z_1 = L$ are unimportant for the further presentation and can be of any kind. For determinacy we take

$$\begin{aligned} &\Theta_3 = \Theta_3^{b}(r, t), \quad 0 < t \leqslant T, \; R_1 \leqslant r \leqslant R_2, \; z_1 = 0, \\ &\Theta_3 = \Theta_3^{e}(r, t), \quad 0 < t \leqslant T, \; R_1 \leqslant r \leqslant R_2, \; z_1 = L, \\ &\Theta_3 = \Theta_3^{0}(r, z_1), \quad t = 0, \; R_1 \leqslant r \leqslant R_2, \; 0 \leqslant z_1 \leqslant L. \end{aligned} \qquad (2)$$

Let us consider the main idea of the algorithm on the example of the semidetailed conjugate problem. The temperature fields in the heat-transfer agents are described by the quasi-one-dimensional, nonsteady energy equation

$$\rho_i \frac{\partial \Theta_i}{\partial t} + q_i \frac{\partial \Theta_i}{\partial z_i} = \sigma_i (\Theta_{wi} - \Theta_i), \quad 0 < t \leqslant T, \; 0 < z_i \leqslant L \qquad (3)$$

with the boundary conditions

$$\Theta_i = \Theta_i^0(z_i), \quad t = 0, \; 0 \leqslant z_i \leqslant L, \qquad (4)$$

$$\Theta_i = \Theta_i^b(t), \quad 0 \leqslant t \leqslant T, \; z_i = 0.$$

Here $i = 1, 2$ while the functions on the right sides of Eqs. (2) and (4) are given, with $\Theta^0{}_i(0) = \Theta^b{}_i(0)$; ρ_i, q_i, and σ_i are constants.

Boundary conditions of the third kind are considered at the surfaces of the wall bathed by the heat-transfer agents, with the temperature of a heat-transfer agent being determined from Eq. (3):

$$(-1)^{i+1}\frac{\partial \Theta_3}{\partial r} = \sigma_i(\Theta_{wi} - \Theta_i),$$
$$0 < t \leqslant T, \quad r = R_i, \quad 0 < z_i \leqslant L. \tag{5}$$

In the region occupied by the dividing wall we introduce the grid $\bar{\omega} = \bar{\omega}_r \times \bar{\omega}_{z_1}$. where

$$\bar{\omega}_r = \{r_n = R_1 + nh_r, \; n = 0, 1, \ldots, N_r, \; h_r = (R_2 - R_1)/N_r\},$$
$$\bar{\omega}_{z_1} = \{z_m^{(1)} = mh_z, \; m = 0, 1, \ldots, N_z, \; h_z = L/N_z\},$$

as well as the time grid

$$\omega_\tau = \{t_j = j\tau, \; t_{j+1/2} = t_j + \tau/2, \quad j = 0, 1, \ldots, j_0, \; \tau = T/j_0\}.$$

We approximate the differential operators on the right side of Eq. (1) by difference analogs:

$$\frac{\partial^2 \Theta_3}{\partial z_1^2} \to \Lambda_{z_1}\Theta^{(3)} = \Theta^{(3)}_{\bar{z}_1 z_1}, \tag{6}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Theta_3}{\partial r}\right) \to \Lambda_r\Theta^{(3)} = \left(\frac{1}{r}\right)(\tilde{r}\Theta^{(3)}_{\bar{r}})_r, \tag{7}$$

$$\tilde{r} = r_{n-1/2} = 0{,}5(r_n + r_{n-1}).$$

We approximate the boundary conditions (5) in the solution of Eq. (1) by the difference scheme [9]

$$(-1)^{i+1}\frac{r_\gamma}{R_i h_r}\Theta^{(3)}_\delta + \Theta^{(3)}_{\bar{z}_1 z_1} - \frac{\sigma_i}{h_r}(\Theta^{(i)}_{(w)} - \Theta^{(i)}) = \Theta^{(3)}_{\bar{t}}, \tag{8}$$

where

$$\gamma = \begin{cases} N_r - 1/2, & i = 2, \\ 1/2, & i = 1, \end{cases} \qquad \delta = \begin{cases} \bar{r}, & i = 2, \\ r, & i = 1. \end{cases}$$

Using (6), (7), and (8), we write the following locally one-dimensional scheme:

$$\Theta^{(3)j+1/2}_{\bar{t}} = \Lambda_r\Theta^{(3)j+1/2}, \quad R_1 \leqslant r_n \leqslant R_2, \quad 0 < z_m^{(1)} < L, \tag{9a}$$

$$\Theta^{(3)j+1/2}_{\bar{t}} = 0, \; r_n = r_0 = R_1, \; 0 < z_m^{(1)} < L, \tag{9b}$$

$$\Theta^{(3)j+1/2}_{\bar{t}} = 0, \; r_n = r_{N_r} = R_2, \; 0 < z_m^{(1)} < L, \tag{9c}$$

$$\Theta^{(3)j+1}_{\bar{t}} = \Lambda_{z_1}\Theta^{(3)j+1}, \; R_1 < r_n < R_2, \; 0 < z_m^{(1)} < L, \tag{10a}$$

$$\Theta^{(3)j+1} = \Theta^{(3)j+1}_{(b)}, \; R_1 < r_n < R_2, \quad z_m^{(1)} = z_0^{(1)} = 0, \tag{10b}$$

$$\Theta^{(3)j+1} = \Theta^{(3)j+1}_{(e)}, \; R_1 < r_n < R_2, \; z_m^{(1)} = z_{N_z}^{(1)} = L, \tag{10c}$$

$$\Theta^{(3)j+1}_{\bar{t}} = \Lambda_{z_1}\Theta^{(3)j+1} - \frac{\sigma_1}{h_r}(\Theta^{(1)j+1}_{(w)} - \Theta^{(1)j+1}) + \frac{r_{1/2}}{R_1 h_r}\Theta^{(3)j+1}_r, \qquad r_n = r_0 = R_1, \; 0 < z_m^{(1)} < L, \tag{11a}$$

$$\Theta^{(3)j+1} = \Theta^{(3)j+1}_{(b)}, \; r_n = r_0 = R_1, \; z_m^{(1)} = z_0^{(1)} = 0, \tag{11b}$$

$$\Theta^{(3)j+1} = \Theta^{(3)j+1}_{(e)}, \; r_n = r_0 = R_1, \; z_m^{(1)} = z_{N_z}^{(1)} = L, \tag{11c}$$

$$\rho_1 \Theta_{\bar t}^{(1)j+1} + q_1 \Theta_{z_1}^{(1)j+1} = \sigma_1 (\Theta_{(w)}^{(1)j+1} - \Theta^{(1)j+1}), \qquad 0 < z_m^{(1)} \leqslant L,$$

$$\Theta^{(1)j+1} = \Theta_{(b)}^{(1)j+1}, \quad z_m^{(1)} = z_0^{(1)} = 0, \tag{11d}$$

$$\Theta_{\bar t}^{(3)j+1} = \Lambda_{z_1} \Theta^{(3)j+1} - \frac{\sigma_2}{h_r} (\Theta_{(w)}^{(2)j+1} - \Theta^{(2)j+1}) - \frac{r_{N_r - 1/2}}{R_2 h_r} \Theta_{\bar r}^{(3)j+1}, \qquad r_n = r_{N_r} = R_2,\ 0 < z_m^{(1)} < L, \tag{11e}$$

$$\Theta^{(3)j+1} = \Theta_{(b)}^{(3)j+1},\ r_n = r_{N_r} = R_2,\ z_m^{(1)} = z_0^{(1)} = 0, \tag{12a}$$

$$\Theta^{(3)j+1} = \Theta_{(e)}^{(3)j+1},\ r_n = r_{N_r} = R_2,\ z_m^{(1)} = z_{N_z}^{(1)} = L, \tag{12b}$$

$$\rho_2 \Theta_{\bar t}^{(2)j+1} + q_2 \Theta_{z_2}^{(2)j+1} = \sigma_2 (\Theta_{(w)}^{(2)j+1} - \Theta^{(2)j+1}), \qquad 0 < z_m^{(2)} \leqslant L, \tag{12c}$$

$$\Theta^{(2)j+1} = \Theta_{(b)}^{(2)j+1},\ z_m^{(2)} = z_0^{(2)} = 0 \tag{12d}$$

with the initial conditions at t = 0

$$\Theta^{(3)} = \Theta_{(0)}^{(3)} (r_n,\ z_m^{(1)}),\ \Theta^{(i)} = \Theta_{(0)}^{(i)} (z_m^{(i)}),\ i = 1, 2.$$

In contrast to the traditional locally one-dimensional scheme [10], here in the boundary conditions at the heat-transfer-agent—wall interface (8), written for the time j + 1, the derivatives with respect to both coordinates are retained, (11a) and (12a) for i = 1 and i = 2, respectively. This allows us to separate the initial difference problem into four difference problems: two for the heat-conduction equation at the time j + 1/2 with respect to the r coordinate [(9a)-(9c)] and at the time j + 1 with respect to the z_1 coordinate [(10a)-(10c)] and two for the energy equations [(11a)-(11e) and (12a)-(12e)] at the time j + 1. One can see that the difference problems at "whole" times are independent of each other and can be solved autonomously. We note that Eqs. (11a) and (12a) coincide in form with the nonsteady heat-balance equation for a solid body for which the thermal resistance in the transverse direction can be neglected in comparison with the longitudinal thermal resistance.

The solutions to the systems of difference equations (9a)-(9c) and (10a)-(10c) can be obtained by the usual trial-run method [10]. As for the systems of difference equations (11a)-(11e) and (12a)-(12e), the usual trial-run method is inapplicable for their solution. This is connected with the fact that for equations (a) of these systems of equations we have a boundary-value problem, while for equations (d) we have a problem with the initial condition (e). The application of iterations brings none of the advantages given by the proposed locally one-dimensional scheme. Therefore, we consider a direct method of solving the systems of equations (11a)-(11e) and (12a)-(12e).

If we change from the variable z_1 to the variable z_2 in Eq. (12a), then the systems of equations under consideration will be identical, since $\Theta_{\bar z_1 z_1}^{(3)} = \Theta_{\bar z_2 z_2}^{(3)}$, and, omitting the indices j + 1 and n, they can be rewritten in the form

$$-A_m \Theta_{m-1}^{(3)} + C_m \Theta_m^{(3)} - B_m \Theta_{m+1}^{(3)} = R_m + G_m \Theta_m^{(i)}, \tag{13a}$$

$$n = \begin{cases} 0, & i = 1, \\ N_r, & i = 2, \end{cases} \quad m = 1, 2, \ldots, N_z - 1,$$

$$\Theta_0^{(3)} = \Theta_{(b)}^{(3)},\ \Theta_{N_z}^{(3)} = \Theta_{(e)}^{(3)}, \tag{13b}$$

$$\Theta_m^{(i)} = D_m^{(i)} \Theta_{m-1}^{(i)} + E_m^{(i)} \Theta_m^{(3)} + S_m^{(i)}, \tag{13c}$$

$$m = 1, 2, \ldots, N_z,$$

$$\Theta_0^{(i)} = \Theta_{(b)}^{(i)}. \tag{13d}$$

$$A_m = B_m = \tau / h_z^2 , \tag{14}$$

$$C_m = 1 + A_m + B_m + \sigma_i \tau / h_r + r_\gamma / (R_i h_r^2),$$

$$R_m = 1 + r_\gamma \Theta^{(3)}_{m,\gamma+(-1)^i/2} / (R_i h_r^2),$$

$$G_m = \sigma_i \tau / h_r, \ \gamma = \begin{cases} N_r - 1/2, & i = 2, \\ 1/2, & i = 1, \end{cases}$$

$\Theta^{(3)}_{m,\gamma+(-1)^i/2}$ being the solution to the system of equations (10) with $n = \gamma + (-1)^i/2$,

$$D_m^{(i)} = (q_i/h_z)/(\rho_i/\tau + q_i/h_z + \sigma_i) > 0,$$

$$E_m^{(i)} = \sigma_i D_m^{(i)} / (q_i/h_z) > 0, \tag{15}$$

$$S_m^{(i)} = (\rho_i \Theta_m^{(i)j} / \tau) D_m^{(i)} / (q_i / h_z) > 0.$$

We assume that the following relations are satisfied at layer m:

$$\Theta_m^{(i)} = \alpha_m \Theta_m^{(3)} + \beta_m, \tag{16a}$$

$$\Theta_m^{(3)} = P_m \Theta_{m+1}^{(3)} + Q_m. \tag{16b}$$

Eliminating $\Theta^{(3)}_m$ from (16a) and (16b), substituting the resulting expression into Eq. (13c), and preliminarily replacing the index m by m + 1, we obtain

$$\Theta_{m+1}^{(i)} = \alpha_{m+1} \Theta_{m+1}^{(3)} + \beta_{m+1}, \tag{17}$$

where

$$\alpha_{m+1} = (D_{m+1}^{(i)} \alpha_m P_m + E_{m+1}^{(i)}); \tag{18a}$$

$$\beta_{m+1} = D_{m+1}^{(i)} (\alpha_m Q_m + \beta_m) + S_{m+1}^{(i)} . \tag{18b}$$

Analyzing Eq. (13a) at layer m + 1, using Eqs. (16b) and (17) we write

$$\Theta_{m+1}^{(3)} = P_{m+1} \Theta_{m+2}^{(3)} + Q_{m+1}, \tag{19}$$

where

$$P_{m+1} = B_{m+1} / (-A_{m+1} P_m - G_{m+1} \alpha_m + C_{m+1}); \tag{20a}$$

$$Q_{m+1} = (R_{m+1} + G_{m+1}\beta_m + A_{m+1}Q_m) P_{m+1}/B_{m+1}. \tag{20b}$$

At layer m = 0, comparing (16a) and (16b) with the boundary conditions (13d) and (13b), respectively, we have

$$\alpha_0 = 0, \ \beta_0 = \Theta_{(b)}^{(i)} , \tag{21a}$$

$$P_0 = 0, \ Q_0 = \Theta_{(b)}^{(3)} . \tag{21b}$$

For boundary conditions different from (13b) the latter relations must be altered appropriately.

A direct trial run by Eqs. (18) and (20) for each time j gives the coefficients of the recurrent relations (16), from which the values of the grid function $\Theta^{(3)}_m$ and $\Theta^{(i)}_m$ ($m = 1, 2, \ldots, N_z - 1$) are found by an inverse trial run. If the coefficients ρ_i, q_i, and σ_i of Eq. (3) are functions of Θ_i, then this method of solving the system of equations (13) must be combined with iterations.

Using the method of [10], one can show that the modified trial-run method described is stable with respect to random errors and is well-posed, since $|P_m| \leq 1$ and $|\alpha_m| \leq 1$ ($m + 0, 1, \ldots, N_z - 1$) and the denominators of Eqs. (20a) differ from zero.

Thus, all four systems of equations are solved by direct (noniterative) methods using the usual or modified trial runs. This means that the locally one-dimensional scheme considered here is economical, i.e., it requires the order of O(1) operations per grid node. As for the accuracy of the scheme, by applying the method of [9] one can show that the scheme (9)-(12) converges uniformly to the exact solution of the initial boundary-value problem (1)-(5) at a rate $O(h_z + h_r + \tau)$.

This method of constructing a locally one-dimensional scheme can also be applied to the solution of the detailed conjugate problem of heat exchange. In this case, instead of Eqs. (2) and the boundary conditions (5), one must consider the nonsteady energy equation

$$a_i \frac{\partial \Theta_i}{\partial t} + c_i \frac{\partial \Theta_i}{\partial z_i} + d_i \frac{\partial \Theta_i}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left(b_i r \frac{\partial \Theta_i}{\partial r} \right) + g_i, \tag{22}$$

$$i = 1,\ 2,\ 0 < t \leqslant T,\ \left.\begin{matrix} i = 1,\ 0 \\ i = 2,\ R_2 \end{matrix}\right\} < r < \left\{\begin{matrix} R_1,\ i = 1, \\ R_3,\ i = 2, \end{matrix}\right.\ 0 < z_i \leqslant L$$

with internal boundary conditions of the fourth kind

$$\Theta_3 = \Theta_i,\ \frac{\partial \Theta_3}{\partial r} = K_s \frac{\partial \Theta_i}{\partial r},$$

$$0 < t \leqslant T,\ r = R_i,\ 0 < z_i < L,\ i = 1,\ 2. \tag{23}$$

The finite-difference approximation of Eq. (22) has the index form

$$-A^{(i)}_{m,n} \Theta^{(i)j+1}_{m,n-1} + C^{(i)}_{m,n} \Theta^{(i)j+1}_{m,n} - B^{(i)}_{m,n} \Theta^{(i)j+1}_{m,n+1} = F^{(i)}_{m,n}, \tag{24}$$

where the free term can be

$$F^{(i)}_{m,n} = G^{(i)}_{m,n} \Theta^{(i)j+1}_{m-1,n} + R^{(i)}_{m,n}.$$

The internal boundary condition is approximated by an expression of the type

$$\Theta^{(3)} = \Theta^{(i)},\ 0 < t_j \leqslant T,\ r_n = R_i,\ 0 < z^{(1)}_m < L,\ i = 1, 2, \tag{25}$$

$$(-1)^{i+1} \frac{r_\gamma}{R_i h_r} \Theta^{(3)}_\delta + \Theta^{(3)}_{\bar{z}_1 z_1} - \frac{K_s}{h_r} \Theta^{(i)}_\varepsilon = \Theta^{(3)}_t,$$

$$\gamma = \left\{\begin{matrix} N_r - 1/2,\ i = 2, \\ 1/2,\quad i = 1, \end{matrix}\right.\ \delta = \left\{\begin{matrix} \bar{r},\ i = 2, \\ r,\ i = 1, \end{matrix}\right.\ \varepsilon = \left\{\begin{matrix} r,\ i = 2, \\ \bar{r},\ i = 1. \end{matrix}\right.$$

Splitting the condition (26) by analogy with the condition (8) [Eqs. (9b) and (11a) for i = 1 and (9c) and (12a) for i = 2] and replacing Eqs. (11a) and (12a) and Eqs. (11d) and (12d) of the locally one-dimensional scheme (9)-(12) by Eqs. (25) and (24), respectively, we obtain a locally one-dimensional scheme for the detailed conjugate problem having the same properties as that considered earlier. A noniterative algorithm for the solution of a system of equations of the type (24)-(25) was proposed in [11]. A proof that it is stable and well-posed is also given there and calculations by this algorithm are presented. Thus, the locally one-dimensional scheme constructed in combination with the algorithm from [11] is also economical and requires O(1) operations per grid node.

In conclusion, we note that the method of splitting the internal boundary condition (25) under consideration makes it possible, in a computer realization of the locally one-dimensional scheme for solving detailed conjugate problems of heat exchange, to use ready-made programs for solving a heat-conduction equation by the locally one-dimensional method and an energy equation of the type (22). Since in the algorithm of [11] it is required to solve a difference equation of the type (24) twice at each layer m, the total time T_c for solving the conjugate problem is approximately equal to $T_c \cong T^{(1)}{}_c + 2nT^{(2)}{}_c$, where $T^{(1)}{}_c$ and $T^{(2)}{}_c$ are the times of solution of the heat-conduction and energy equations, respectively, while n = 1 for one-sided heating and n = 2 for two-sided heating in concurrent flow or counterflow.

NOTATION

Θ_i, $\Theta^{(i)}$, dimensionless temperatures of heat-transfer agents (i = 1, 2) and the wall (i = 3) and their grid analogs; Θ_{wi}, $\Theta^{(i)}{}_{(w)}$, temperature of the wall surface at $r = R_i$ and its grid analog; $\Theta^b{}_3$, $\Theta^{(3)}{}_{(b)}$, temperature of the wall surface at $z_1 = 0$ (beginning) and its grid analog; $\Theta^e{}_3$, $\Theta^{(3)}{}_{(e)}$, temperature of the wall surface at $z_1 = L$ (end) and its grid analog; $\Theta^0{}_i$, $\Theta^{(i)}{}_{(0)}$, temperatures of the heat-transfer agents (i = 1, 2) and the wall (i = 3) at t = 0 and their grid analogs; $\Theta^b{}_i$, $\Theta^{(i)}{}_{(b)}$, temperatures of the heat-transfer agents (i = 1, 2) at the channel entrance ($z_i = 0$) and their grid analogs; $\Theta^j{}_{m,n}$ (Θ_m), index designations of the grid analogs of the temperatures introduced in (24) and (13); t, dimensionless time; r, z_i, dimensionless spatial coordinates; R_i, radii of the inner (i = 1) and outer (i = 2) surfaces of the wall and of the outer (i = 3) channel; L, wall length; σ_i, dimensionless heat-transfer coefficient; K_S, conjugation number; h_r, h_z, τ, steps of the space–time grid in the directions r, z_1, and t, respectively; m, n, j, coordinates of nodes of the space–time grid; $\Theta_r = (\Theta_{n+1} - \Theta_n)/h_r$; $\Theta_{\bar{r}} = (\Theta_n - \Theta_{n-1})/h_r$; $\Theta_{\bar{r}r} = (\Theta_r - \Theta_{\bar{r}})/h_r$.

LITERATURE CITED

1. Inverse and Conjugate Problems of Heat Exchange (Materials to the session of the section Methods of Calculation of Processes and Physicochemical Transitions in High-Temperature Materials) [in Russian], Moscow (1979), p. 43.
2. R. S. Levitin, "On the efficiency of the use of new methods of solving conjugate problems in heat-exchange apparatus and constructions," Vestsi Akad. Navuk BSSR, Ser. Fiz.-Energ. Navuk, No. 3, 111-116 (1981).
3. A. Sh. Dorfman, "Some properties of the temperature distributions of the interfaces in heat exchange between a plate and liquids flowing over it," Teplofiz. Vys. Temp., 13, No. 1, 116-120 (1975).
4. D. Wérde, "Verhalten von Gleichstrom-Doppelrohv-Wärmeaustauschern bei turbulenter Strömung unter besonderer Berücksichtingung der axialen Wärmeleitung in der einzelnen Systembereichen," Ber. Inst. Kerntech. Tech. Univ. Berlin, No. 58 (1976).
5. V. S. Ryaben'kii, "Method of internal boundary conditions in the theory of difference boundary-value problems," Usp. Mat. Nauk, 26, No. 3(159), 106-160 (1971).
6. S. Mory, M. Kataya, and A. Tanimoto, "Performance of counterflow parallel plate heat exchangers under laminar flow conditions," Heat Transfer Eng., 2, No. 1, 28-38 (1980).
7. V. M. Bregman and B. M. Mitin, "On the approximate solution of the conjugate problem of convective heat exchange for a plane wall," Tr. Tsentr. Nauchno-Issled. Inst. Aviamotorostr., No. 1060 (1983).
8. R. P. Stein, "Mathematical and practical aspects of heat transfer in double pipe heat exchangers," Proceedings of the Third International Heat Transfer Conference, AIChE, New York (1966), pp. 139-148.
9. T. S. Efremova and I. V. Fryazinov, "Economical schemes for one version of the third boundary-value problem," Zh. Vychisl. Mat. Mat. Fiz., 13, No. 2, 356-364 (1973).
10. A. A. Samarskii, Theory of Difference Schemes [in Russian], Nauka, Moscow (1977).
11. V. V. Sapelkin, "Conjugate problem of nonsteady heat exchange between a laminar boundary layer and a plate with internal heat sources," Teplofiz. Vys. Temp., 19, No. 6, 1213-1220 (1981).